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Quantum State Transfer via Parity Measurement

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Abstract We propose two schemes for quantum state transfer using parity measurement in a cavity-waveguide system, and the two schemes can be generalized to multidipole's case. An important advantage is that quantum state transfer can be completed by single-qubit rotations and the measurement of parity. Therefore, our scheme can be realized in the scope of current experimental technology.

Keywords Single-qubit rotation · Parity measurement · Single-qubit measurement

Quantum information science has made rapid progress in recent years. Quantum teleportation is an indispensable essential component for many applications of quantum information. Since the first create of quantum teleportation scheme proposed by Bennett et al. [1], there have been wide investigations on this field both in theories [2–5] and in experiments [6, 7]. On the other hand, the cavity quantum electrodynamics (QED) system is another qualified candidate for demonstrating quantum information processing. Davidovich et al. [8] have presented a scheme which is used to teleport an unknown atomic state between two high-Q cavities. Cirac et al. [9] have made another cavity QED proposal for the realization of quantum teleportation of an atomic state by using two additional atomic levels of one of the correlated pair. In 2004, Zheng [10] proposed a novel scheme for teleporting an unknown atomic state by two atoms interacting with a single-mode cavity field.

However, different from quantum teleportation, quantum state transfer only retains the part of teleportation which is necessary for computing. Quantum state transfer cannot replace teleportation in non-local applications, but quantum state transfer needs less measurements and less auxiliary qubits than teleportation. So efficient short-distance quantum state transfer is also an important problem in the field of quantum computing. Yin et al. [11] propose a quantum state transfer scheme between two remote cavities via an optical fiber. Recently, Waks et al. [12] show an interesting effect of dipole induced transparency (DIT) that

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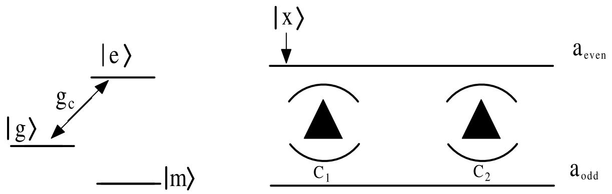

Fig. 1 Schematic setup for transferring a quantum state from the dipole 1 to 2 by measuring the parity of the dipoles in C_1 and C_2. Each dipole has three relevant energy level $|g\rangle$, $|m\rangle$ and $|e\rangle$. The cavity mode interacts with the transition between $|g\rangle$ and $|e\rangle$ resonantly

a dipole is placed in a drop-filter cavity, the waveguide becomes highly transparent when the vacuum Rabi frequency of the dipole g is much smaller than the cavity decay, and generate Bell states using parity measurement. Based on this kind of method, Qian et al. [13] propose a scheme to generate an arbitrary multi-qubit Greenberger-Horne-Zeilinger states by parity measurement. We are inspired by these works of [10–13], so we present two simplification schemes for quantum state transfer based on dipole-induced transparency in a cavity-waveguide system, and generalize the two schemes to multidipole's case.

Now let us depict our scheme for quantum state transfer of one-particle. We show the schematic setups in Fig. 1. There are two cavities which contain a single dipole emitter, respectively, and they are evanescently coupled to up and down waveguides. Each dipole has three relevant energy levels: a ground state $|g\rangle$, an excited state $|e\rangle$, and a metastable state $|m\rangle$. The parity of the dipoles can be checked via the photon detectors a_{even} and a_{odd}. The purpose of this scheme is to transfer the state $|\phi\rangle = \alpha|g\rangle + \beta|m\rangle$ from the dipole 1 to 2. Here $|\alpha|^2 + |\beta|^2 = 1$, and the total state of the two dipoles is

$$(\alpha|g\rangle + \beta|m\rangle)_1(|g\rangle + |m\rangle)_2 = \alpha(|g,g\rangle + |g,m\rangle)_{1,2} + \beta(|m,g\rangle + |m,m\rangle)_{1,2}, \tag{1}$$

where subscripts 1 and 2 on the states denote for the dipole 1 and 2, respectively. For convenience, we omit the normalization factors here and in what follows. Now we inject a probe field $|\mathbf{x}\rangle$ into the waveguide before the cavity C_1, and then check the parity of the two dipoles 1 and 2. We can conclude immediately that the four terms of the two dipoles will deduce to two terms of them. We will obtain the state $\alpha|g,g\rangle + \beta|m,m\rangle$ if the parity of the dipoles is found in the even-parity state (a_{even}). On the other hand, the total state collapses to $\alpha|g,m\rangle + \beta|m,g\rangle$ if the parity is found in the odd-parity state (a_{odd}). We then perform a Hadamard gate operation H on the first dipole. The operation H transforms states as $|g\rangle \to \frac{1}{\sqrt{2}}(|g\rangle + |m\rangle)$ and $|m\rangle \to \frac{1}{\sqrt{2}}(|g\rangle - |m\rangle)$. After making a Hadamard gate on the first dipole, the state of system can be expressed as

$$|\Phi\rangle \to \begin{cases} |g\rangle(\alpha|g\rangle + \beta|m\rangle) + |m\rangle(\alpha|g\rangle - \beta|m\rangle), & a_{\text{even}}, \\ |g\rangle(\alpha|m\rangle + \beta|g\rangle) + |m\rangle(\alpha|m\rangle - \beta|g\rangle), & a_{\text{odd}}, \end{cases} \tag{2}$$

then we performs single-qubit measurement on the dipole 1. According to the measurement result and the parity of the two dipoles, we can make sure that the second dipole will collapse to a deterministic state. As a result, the unknown state of the dipole 1 is transformed to the second dipole after performing one of four unitary operations. The four unitary operations

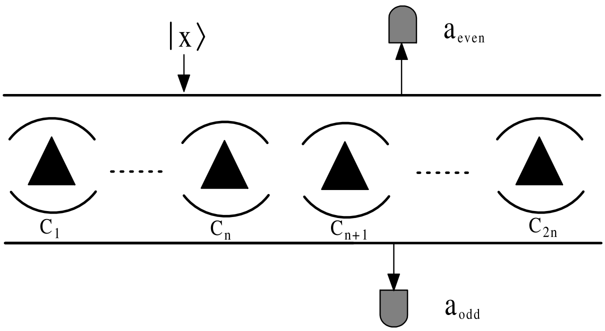

Fig. 2 Transferring an n-particle state. For the measurement of parity, the mirrors guide the probe field $|\mathbf{x}\rangle$ into the detector a_{even} or a_{odd}

can be as follows:

$$U_{00}=\begin{pmatrix}1&0\\0&1\end{pmatrix},\qquad U_{01}=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$
$$U_{10}=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\qquad U_{11}=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \tag{3}$$

For example, when the state of the first dipole is in $|m\rangle$ and the two dipoles's parity is odd, the quantum state transfer can be done by performing the operation U_{10} as follows: Excite $|g\rangle\to|m\rangle$, $|m\rangle\to-|g\rangle$.

Furthermore, as shown in Fig. 2, we can generalize this method to multidipole's example. Suppose that an unknown n-particle state of general form that will be transformed is $\alpha\prod_{j=1}^{n}|g_j\rangle+\beta\prod_{j=1}^{n}|m_j\rangle$, the target state is in the following state $\prod_{i=n+1}^{2n}(|g_i\rangle+|m_i\rangle)$, similar to the above case, we input a probe field to measure the parity of the neighboring dipoles (n and $n+1$). The system's states can be converted into

$$|\Phi\rangle\to\begin{cases}\alpha\prod_{j=1}^{n}|g_j\rangle\prod_{i=n+1}^{2n}|g_i\rangle+\beta\prod_{j=1}^{n}|m_j\rangle\prod_{i=n+1}^{2n}|m_i\rangle, & a_{\text{even}},\\ \alpha\prod_{j=1}^{n}|g_j\rangle\prod_{i=n+1}^{2n}|m_i\rangle+\beta\prod_{j=1}^{n}|m_j\rangle\prod_{i=n+1}^{2n}|g_i\rangle, & a_{\text{odd}}.\end{cases} \tag{4}$$

Based on the same method, perform Hadamard gate operations on the dipoles a_1–a_n, then we perform single-qubit measurements on a_1–a_n. Obviously the unknown state is transformed to the dipoles b_1–b_n after performing n unitary operations.

Then, we consider the transfer of an arbitrary two-qubit state using dipole-induced transparency. Suppose that the two transformed dipoles (1 and 2) are in an unknown state

$$|\chi\rangle=\alpha|g,g\rangle+\beta|g,m\rangle+\gamma|m,g\rangle+\delta|m,m\rangle, \tag{5}$$

where α, β, γ and δ are arbitrary complex numbers, and satisfy normalization conditions $|\alpha|^2+|\beta|^2+|\gamma|^2+|\delta|^2=1$. The setup is depicted in Fig. 3. The two target dipoles (3 and 4) are in the state $|g\rangle+|m\rangle$. The state of whole system can be described by

$$\begin{aligned}&(\alpha|g,g\rangle+\beta|g,m\rangle+\gamma|m,g\rangle+\delta|m,m\rangle)_{1,2}(|g\rangle+|m\rangle)_3(|g\rangle+|m\rangle)_4\\ &\quad=[\alpha|g,g\rangle(|g,g\rangle+|g,m\rangle+|m,g\rangle+|m,m\rangle)\\ &\qquad+\beta|g,m\rangle(|g,g\rangle+|g,m\rangle+|m,g\rangle+|m,m\rangle)\\ &\qquad+\gamma|m,g\rangle(|g,g\rangle+|g,m\rangle+|m,g\rangle+|m,m\rangle)\\ &\qquad+\delta|m,m\rangle(|g,g\rangle+|g,m\rangle+|m,g\rangle+|m,m\rangle)]_{1,2,3,4}.\end{aligned} \tag{6}$$

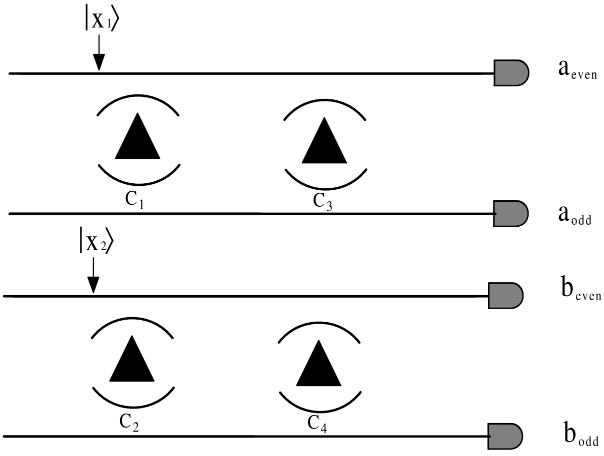

Fig. 3 Transferring an arbitrary two-particle state from the dipoles 1 and 2 to 3 and 4

The four dipoles are placed in C_1, C_2, C_3 and C_4, respectively. Then we input two probe beams $|\mathbf{x}_1\rangle$ and $|\mathbf{x}_2\rangle$ before the cavities C_1 and C_2, respectively, and measure the parities of the four dipoles. The four possible results are given by

$$|\Psi\rangle \to \begin{cases} (\alpha|g,g,g,g\rangle + \beta|g,m,g,m\rangle + \gamma|m,g,m,g\rangle \\ \quad + \delta|m,m,m,m\rangle)_{1,2,3,4} & (a_{\text{even}},\ b_{\text{even}}), \\ (\alpha|g,g,m,g\rangle + \beta|g,m,m,m\rangle + \gamma|m,g,g,g\rangle \\ \quad + \delta|m,m,g,m\rangle)_{1,2,3,4} & (a_{\text{odd}},\ b_{\text{even}}), \\ (\alpha|g,g,g,m\rangle + \beta|g,m,g,g\rangle + \gamma|m,g,m,m\rangle \\ \quad + \delta|m,m,m,g\rangle)_{1,2,3,4} & (a_{\text{even}},\ b_{\text{odd}}), \\ (\alpha|g,g,m,m\rangle + \beta|g,m,m,g\rangle + \gamma|m,g,g,m\rangle \\ \quad + \delta|m,m,g,g\rangle)_{1,2,3,4} & (a_{\text{odd}},\ b_{\text{odd}}). \end{cases} \tag{7}$$

We perform two Hadamard operations on the dipoles 1 and 2 and make single-qubit measurements on them. According to the measurement results and the parities of four dipoles, we can transfer the arbitrary two-qubit state to the other two dipoles (3 and 4) by two of four single-qubit operations (see Table 1).

Table 1 Corresponding relations among the parities of the four dipoles, the measurement results of the two dipoles (1 and 2), and performing two unitary operations on the other two dipoles (3 and 4). Here the superscripts 3 and 4 denote the two dipoles 3 and 4

	a_{even}	b_{even}	a_{odd}	b_{even}	a_{even}	b_{odd}	a_{odd}	b_{odd}
$\|g,g\rangle$	U_{00}^3	U_{00}^4	U_{01}^3	U_{00}^4	U_{00}^3	U_{01}^4	U_{01}^3	U_{01}^4
$\|g,m\rangle$	U_{00}^3	U_{11}^4	U_{01}^3	U_{11}^4	U_{00}^3	U_{10}^4	U_{01}^3	U_{10}^4
$\|m,g\rangle$	U_{11}^3	U_{00}^4	U_{10}^3	U_{00}^4	U_{11}^3	U_{01}^4	U_{10}^3	U_{01}^4
$\|m,m\rangle$	U_{11}^3	U_{11}^4	U_{10}^3	U_{11}^4	U_{11}^3	U_{10}^4	U_{10}^3	U_{10}^4

By adding more dipoles and cavities in Fig. 3, the above idea is in principle extendable to an arbitrary n-qubit case. Supposing we want to transfer an unknown arbitrary n-qubit state

$$\begin{aligned}|\phi_n\rangle &= \sum_{i=1}^{2^n}\alpha_i|\chi_i^1,\chi_i^2,\ldots,\chi_i^n\rangle\\ &= \alpha_1|g,g,\ldots,g\rangle+\alpha_2|g,g,\ldots,m\rangle+\cdots+\alpha_n|m,m,\ldots,m\rangle,\end{aligned} \tag{8}$$

and using n unentangled dipoles $|\psi_n\rangle=\prod_{j=n+1}^{2n}(|g\rangle+|m\rangle)_j$ as the target particles. The state of the joint system can be written as

$$|\phi_n\rangle = \sum_{i=1}^{2^n}\alpha_i|\chi_i^1,\chi_i^2,\ldots,\chi_i^n\rangle\otimes\prod_{j=n+1}^{2n}(|g\rangle+|m\rangle)_j. \tag{9}$$

Then we inject n probe beams before the cavities $(C_1,C_2,\ldots,C_n)$, respectively, and measure the parities of the $2n$ dipoles. So we complete the transfer of an arbitrary n-qubit state by performing n unitary operations on the other n dipoles (from $n+1$ to $2n$) after making H operations and single-qubit measurements on these dipoles (from 1 to n). According to Table 1, we can get the generally expression of unitary operation $U^n_{i,j\oplus i}$. Here superscript n denotes the dipole n, subscript i corresponds to single-qubit measurement result $|g\rangle$ and $|m\rangle$ ($|g\rangle=0$ and $|m\rangle=1$), subscript j corresponds to the parity ($a_{\mathrm{even}}=0$ and $a_{\mathrm{odd}}=1$), and "$\oplus$" is performed modulo 2. For example, if the measurement results are $|g\rangle$ and a_{odd} for the dipole n, we perform the unitary operation $U^n_{0,1}$.

We now give a brief discussion on the experimental realization of the proposed scheme based on dipole-induced transparency in a cavity-waveguide system. Our scheme proposed here requires (1) parity measurement of dipoles, (2) single-qubit rotation operations, (3) making single-qubit measurement. Firstly, Waks et al. [12] show that two dipoles which are placed in a drop-filter cavity, respectively, the entanglement between two dipoles can be generated by performing a nondestructive parity measurement. When the energy decay rate from the cavity into each waveguide is between approximately 0.6 and 10 (THz), the successful probability is greater than 99%. The analytic results can low the constraint on using the high-Q regime in our scheme. Secondly, an arbitrary single-qubit rotations can be realized by applying laser field and cavity mode that satisfy the Raman-resonance condition in [14]. Thirdly, we can prepare a dipole in the state $|m\rangle$ as one ancilla to perform single-qubit measurement. The ancilla is also placed in a cavity-waveguide system. We input a probe field to measure the parity of the ancilla and the measured dipole. If the dipole's parity is odd (even), the measured dipole is in the state $|g\rangle$ ($|m\rangle$). Furthermore, similar to [13], the detunings between two optical fields and the corresponding dipole transition are large enough. The three-level system is equivalent to the two-level qubit between the metastable states $|m\rangle$ and $|g\rangle$. So the influence of spontaneous emission is alleviated. Therefore, we believe the proposed scheme might be realized within the reach of the current experimental technology.

In summary, we have proposed how to transfer a single-qubit state and an arbitrary two-qubit state based on QIT, and shown that the two schemes can be generalized to multidipole's example. Different from the previous methods [8–11], we complete the task of transferring quantum state by performing parity check and single-qubit rotations. We hope that with the existing technology it may be possible to implement the quantum state transfer protocol with ease.

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